Optimization Analysis of Satellite-Based ICBM Interceptor Systems

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With the criterion being to minimize the total in-orbit weight of a satellite-based anti-ICBM interceptor concept, this system analysis shows that definite optimizations for the values of the interceptor and system design parameters exist. General, analytical parametric solutions are developed and presented for these optimizations. The resultant relationships permit determination of the optimum values for interceptor range, interceptor characteristic velocity, satellite deployment pattern, total number of satellites in orbit, and total system weight in orbit. These results are presented as a function of interceptor propulsion parameters, ICBM boost characteristics, interceptor acceleration profile, interceptor guidance efficiency, available interception time, target-prediction capability, and ICBM dispersal patterns and densities. Using the data presented, the net effect of the variation of any one parameter or combination of parameters can be found, since the optimum system design can be determined readily as a function of the varied parameters. Most of the results are presented graphically in nondimensional parametric form for maximum utility. Appropriate cases are illustrated in numerical form.

Nomenclature

r_{rel} or r = maximum (slant) range of interceptor relative to satellite
x_{rel} or x = maximum horizontal range relative to satellite
h_{rel} or h = maximum altitude travel relative to satellite
R_t = horizontal range of target prior to interception
Λ = prediction coefficient, defined by Eq. (1)
d_c = diameter of satellite coverage domain
W_0 = gross weight of interceptor
W_{pay} = payload weight of interceptor (includes fixed weight of guidance and control system)
μ = rocket mass ratio, gross weight per pound of payload weight
η = structural deficiency, ratio of variable nonfuel weight to fuel weight of interceptor; includes structural weight, propulsion system weight, variable guidance and control system weight
V_{ch} = characteristic velocity of interceptor
I_{sp} = specific impulse of fuel
δ = percent of satellite empty weight which is proportional to the total weight of interceptors
p = number of interceptors per satellite
ψ_1 = defined in Eq. (21)
ψ_2 = defined in Eq. (21)
ϕ = defined in Eq. (21)
α = interceptor acceleration profile factor, defined by Eq. (12)
ν = measure of guidance efficiency, defined in Eq. (13)
g = acceleration of gravity at earth's surface
t_f = maximum time of interceptor flight
d_{long} = longitudinal spacing between satellites in the same orbit
d_{lat} = lateral spacing between orbits at the critical latitude
λ_{cr} = critical latitude, minimum latitude of complete coverage by AICBM system
d_{orb} = spacing between satellites
N_{th} = theoretical minimum number of satellites, defined by Eq. (3)
ξ = network efficiency factor, defined by Eq. (4)
m = number of satellite orbits
n = number of satellites per orbit
N = total number of satellites in orbit, in a given system
R_p = radial distance from center of earth to the spacing pattern
W_{sys} = total weight of entire orbital AICBM system
W_{sat} = weight of single satellite (including interceptors)
W_{s0} = fixed weight portion of satellite mass [defined in Eq. (18)]
$(1 + \Omega)$ = ratio of actual practical-staged vehicle weight to infinite-staged vehicle weight

I. Introduction

A. Purpose

THE purpose of the investigation is to perform parametric analyses of satellite-based ICBM interceptor system concepts in order to determine the design parameters that will minimize the total system in-orbit weight under a variety of system assumptions and constraints. In these analyses it was determined that definite optimizations for the values of the interceptor and system parameters do exist, and that the total weight in orbit is fairly sensitive to the utilization of the optimum values.

B. System Concept

To facilitate understanding of the fundamentals of the analyses, the system concept under consideration will be described briefly. The concept is directed toward the destruction of enemy ICBM vehicles during their boost period, by intercepting AICBM's to be launched from satellites passing over potential launch areas. This concept has a number of important practical advantages, but an evaluation of the practical and political merits of the various potential anti-ICBM systems is well beyond the scope of the present paper.

To have interceptor vehicles in sufficiently close proximity to the potential ICBM launch sites, a satellite coverage system is used. Assuming that a worldwide network of satellites exists, the system must be designed so that there always will be one satellite within suitable range of every suspected ICBM launch site. Figure 1 is a pictorial diagram of a segment of the system. Two satellites in orbit and the coverage domain of one are shown. Illustrated are the regions vulnerable to an interceptor in the satellite at this instant and the launch regions on the earth's surface which the satellite can cover. If a suitable means of determining the satellite coverage domain can be obtained, it is necessary only to space

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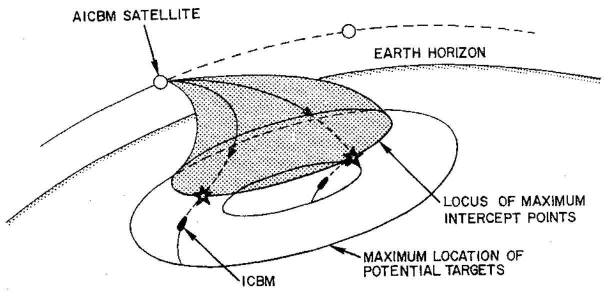

Fig. 1 Concept of a satellite-based interceptor

the proper number of satellites in orbit and to place the orbits sufficiently close together so that the domains of responsibility overlap to give complete coverage of the earth in the area in question.

In the analyses in this paper, it is assumed that only the minimum number of satellites required to give complete coverage without purposeful redundancy will be used. It is clear, however, that, if an optimum design is determined for a nonredundant coverage system, the best way to obtain one or two degrees of redundancy, for example, simply is to use two or three complete systems identical to the optimized nonredundant design. Thus, for any desired degree of redundancy, the system developed here simply would be duplicated the appropriate number of times.

This study also assumes a regular pattern of satellites, i.e., one maintained in a specific desired orbital location. If random satellite patterns are desired instead, the optimum regular pattern is the reference that should be used to develop an efficient random pattern while maintaining a minimum system weight. A random pattern, of course, will require a greater number of interceptors in orbit, as compared to a regular pattern for an equivalent minimum degree of coverage.

The interception concept, then, is to have satellites capable of sensing an enemy ICBM being launched, to provide firing of an interceptor toward that ICBM, and to have that interceptor capable of attacking and destroying the ICBM before the end of the ICBM's boost period. The requirement for attacking before the end of the boost period is a severe constraint and inherently requires high accelerations and velocities of the interceptor.

C. Fundamental Considerations

By examining Fig. 2 (satellite view of Fig. 1), the pertinent domains of coverage may be determined qualitatively. Three circular areas are illustrated. The first circular area of interest is located at the lower extreme of the funnel-shaped domain of the satellite vehicle (see Fig. 1). This is labeled A. This circle illustrates the maximum range of the interceptor vehicle at the lowest possible altitude of interception. Thus, this particular satellite would be unable to attack any ICBM that does not arrive, by the maximum time of interception, within the region illustrated.

The largest circle B is located on the surface of the earth. This indicates the maximum location that an ICBM vehicle possibly could have at the time of launch in order to enter the satellite's effective envelope (region A) by the required time of interception. It is clear that no potential targets (with respect to this particular satellite) can exist outside this circle. On the other hand, there is no assurance that a given ICBM launched within this circle ultimately will enter the domain of the interceptor.

The innermost circle D, however, illustrates the launch region from which all launched ICBM's will have the property of remaining within the interceptor's envelope during the maximum time permitted for interception. Thus, the innermost circular area represents the region in which all launched ICBM's can be attacked successfully by this satellite. The outer circle indicates the area beyond which no launched ICBM could be attacked successfully. The intervening region (D-A) contains a mixture of targets that may or may not be attacked successfully, depending upon their direction of launch. If the satellite attempts to attack potential targets within this region, unsuccessful ICBM launches will be made unless the satellite uses some form of pre launch fire-control prediction to decide which potential targets within this region to attack.

1. *Target prediction*

The first significant criterion imposed on the system now is obvious. It is theoretically possible to design the system with an efficient homing guidance that would attack successfully all vehicles launched within the inner circle with no need for the use of fire-control prediction. If, however, the satellites are more widely spaced to take advantage of the larger area in the interceptor envelope circle, some form of prediction for fire control must be used to assign each target to the appropriate satellite. The use, or lack of use, of prediction in the system is therefore an important parameter in these analyses.

2. *Number of interceptors required*

A second important parameter for the systems designer is that of determining how many interceptors should be carried aboard each satellite vehicle. The choice will depend logically upon the distribution and density of the potential launch sites of the ICBM. In this investigation, two extreme possibilities are assumed; the first assumption is that the targets are distributed nonuniformly and clumped into groups, which in turn are scattered in random fashion throughout the overall target area. The alternate assumption is that the targets are distributed homogeneously throughout the overall target area and that the launch times will be distributed in a random fashion. The implication of these assumptions is quite different and is discussed partially below.

In the clumped-target investigation, the area diameter of the clumps of targets is assumed to be small with respect to the diameter of the coverage circle of the interceptors, and the spacing between clumps is assumed large with respect to the diameter of the interceptor's coverage domain. It is therefore proper to assume that each satellite must carry enough interceptors to attack the number of target ICBM's which can be expected from one of these clumps (during the

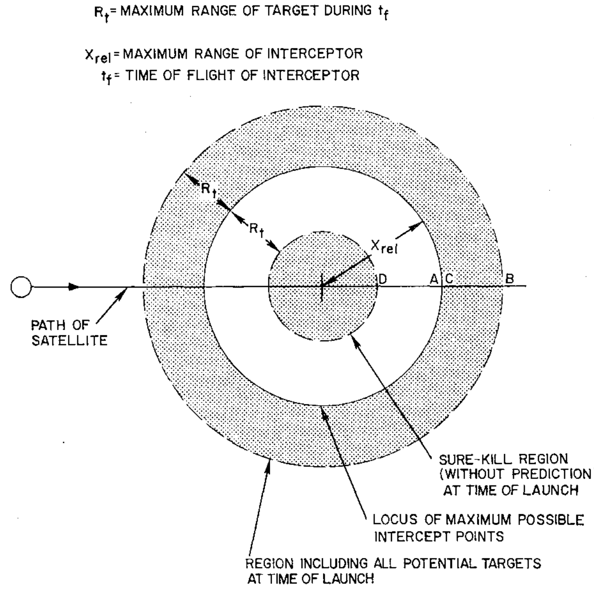

Fig. 2 Satellite coverage domains

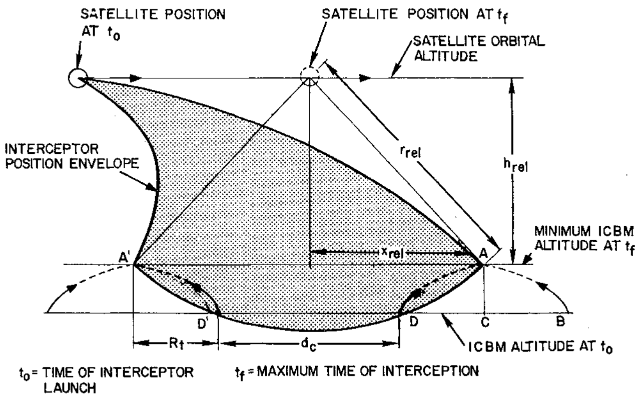

Fig. 3 Coverage domain relationships

time in which that clump lies within the coverage domain of that satellite). Actually, the satellite would not be limited necessarily to one-clump capability but could be designed to handle two or more clumps of vehicles. The specific assumption made here, however, is that the number of ICBM's which the satellite is designed to handle is not a function of the range of the interceptor vehicle. In the extreme, this assumption is, of course, unreasonable. Recall, however, that the spacing between clumps is assumed large with respect to the diameter of responsibility d_c of the satellite. Therefore, if, in using this assumption, the optimum diameter of responsibility is, in fact, small with respect to the expected spacing between clumps, the assumption can be considered valid. Numerical values determined from results later in this paper suggest that the assumption is reasonable.

Now consider the alternative target distribution assumption. If the targets are assumed to be distributed homogeneously and to be launched in a random manner with respect to time, the number of interceptors aboard each satellite must be proportional directly to the area of coverage. This conclusion was used in the second portion of the analyses to compare the effects of the two distributions. Of course, some criterion lying between these two extremes may be the most reasonable (e.g., that the number of targets should increase in proportion to the diameters to the one-half power), but the two cases investigated are by far the most amenable to analytical analysis and represent practical limiting cases from which intervening cases can be estimated. It should be pointed out that the optimization does not depend actually upon the target distribution, per se, but simply upon the logic used in assigning the number of interceptors per satellite in an actual system design.

3. *Interceptor acceleration profile*

A third major parameter is the acceleration profile of the interceptor vehicle. The two extreme cases that are reasonable to consider are 1) a vehicle with nominally constant and continuous acceleration, and 2) one that uses only an initial impulse to obtain its velocity. These cases are the extremes, and judgment must be used to determine realistic cases existing between the two extremes. In particular, the purely impulsive interceptor is unrealistic because some corrective capability, at least near the time of interception, always must be expected. It was not necessary in the analyses, however, to consider the acceleration parameter as only these two discrete cases; instead, the effect of any selected acceleration profile can be determined numerically directly from the results.

4. *Other parameters*

Other parameters used in the investigation are propulsion parameters for the interceptor vehicles, weight of the vehicle structure and guidance and control system, weight of the payload, the maximum allowable time for interception, efficiency of the guidance system, altitude of the satellite pattern, and target vehicle characteristics. All of these factors are used as variable parameters in the study.

One important criterion to be recognized is that the design developed for the system depends strongly upon the particular design of the target ICBM. In order to optimize the system to handle a variety of targets, it is necessary to choose the most difficult target expected to be encountered (and attacked with consistent success) as the criterion for optimizing the system.

II. Analyses, Optimizations, and Results

A. Introduction

Direct analyses and optimization analyses of the AICBM system are developed in this section, and their results are presented graphically in parametric form.

B. General Analysis: Any Target Distribution

Analyses in Sec. IIB are general and are applicable to all system concepts considered regardless of target distribution (homogeneous or clumped) or prediction assumptions (limited or perfect prediction).

1. *Coverage domains*

In the previous section, Fig. 2 illustrated the three domains of potential target areas for each satellite. The determination of the analytical relationships describing these regions can be obtained from the geometrical relationships presented in Fig. 3. Location of the satellite orbit and the minimum altitude at which interception is assumed to occur are indicated in Fig. 3.

It should be noted that coverage patterns are set by the minimum altitude at which successful interceptions can be expected. Examination can show that interception at any greater altitude always can be accomplished if the satellite achieves complete coverage at the minimum altitude. In a specific system design, the determination of the expected minimum altitude must be made; quite conservatively, the minimum interception altitude could be assumed to be zero. However, this assumption would seem to be overly cautious; ICBM end-boost can be expected at some reasonable altitude above the high-drag region of the atmosphere.

The various interceptor-domain relationships are more straightforward if one examines them in a moving coordinate system whose origin corresponds to the position of the orbiting satellite. Some of these relationships are indicated in Fig. 3, where the limiting relative positions of the satellite, interceptors, and target are indicated (at time t_f, the maximum allowable time of interceptor flight). Positions A,B,C, and D follow the same nomenclature used in Fig. 2. The distance $\overline{BC}$ (or $\overline{CD}$) is the maximum horizontal ICBM range from the time of interceptor launch (t_0) to the maximum allowable interception time (t_f); the magnitude of the distance $\overline{BC}$ is described as R_t in the following relations. From the forementioned relationships, an analytical description of the domain of coverage can be obtained.

The diameter of the actual satellite coverage domain is given by the following relationship; the quantity Λ is a quantitative measure of the degree of prediction used. It is important to remember that assumptions on prediction here relate only to its use for fire control (or satellite assignment) purposes:

$$d_c = 2(x_{\text{rel}} - \Lambda R_t) \tag{1}$$

where $\Lambda = 0$ for perfect prediction, $0 < \Lambda < 1$ for imperfect prediction, and $\Lambda = 1$ for no prediction.

Examination of the relationships will show that the term Λ is adequate to describe the range of prediction possible from no-prediction through limited-prediction to perfect-prediction. Thus, Λ may vary from zero to one. When perfect prediction is used, Λ is zero and the satellites can be spaced at distances corresponding to the diameter of the interceptor envelope at the interception altitude ($\overline{AA'}$ in Fig. 3). When, however, no prediction whatsoever is used, Λ is one, and the satellites must be spaced at a distance corresponding to the inner sure-kill ($\overline{DD'}$) circle.

Substituting appropriately in Eq. (1) for x_{rel}, the relationship describing the diameter of the coverage domain is obtained as a function of interceptor maximum relative range, maximum vertical descent, fire-control prediction capability, and ICBM horizontal range:

$$d_c = 2\{[r_{\text{rel}}^2 - h_{\text{rel}}^2]^{1/2} - \Lambda R_t\} \tag{2}$$

For convenience, the "rel" subscripts are dropped in the remainder of this paper. It should be emphasized, however, that these quantities *are* relative measurements and not absolute measurements of range and altitude.

2. *Satellite orbital networks*

Having determined the circular domain of coverage for a given satellite, the problem remains to determine an appropriate satellite orbital network and corresponding fire control pattern for each satellite. Although the development of an efficient network and control pattern is important in reducing overall system weight, it is significant to realize that the fundamental interceptor optimization is uncoupled from the satellite network design so that each may be considered independently.

Fundamentally, the developed interceptor optimization will provide the greatest area of coverage per pound of interceptor, regardless of the manner in which the areal domains of coverage are combined. The effect of different networks, therefore, will be only that of changing the constant multiplier on the overall system weight, so long as a uniform and consistent network and control pattern are used.

Under these circumstances the discussion of orbital networks in this paper is limited to two cases. These, nevertheless, should provide both a basic understanding of the network considerations and useful quantitative relationships. The two cases are 1) a hypothetical ideal system, which is quite useful as a reference, and 2) a simple yet relatively efficient system that has practical potential. This single area of orbital networks obviously is worthy of separate dissertations, and several known studies are referenced later in this paper.

The goal of the network design should be a system that gives complete interceptor coverage over the potential target areas while minimizing the overlap between satellite coverage domains. A number of criteria can be used for defining the desired areas of coverage. As a simple example, this investigation assumes that 100% coverage is desired in all regions above a given critical latitude. Thus, the system might be designed to provide complete coverage at all latitudes greater than ±20°. The equations developed are generalized, however, and complete earth coverage can be obtained simply by setting the critical latitude to be 0°.

It is apparent that the theoretical ideal is a purely hypothetical system that can provide complete coverage over the required latitudes while allowing absolutely no overlap between the circular coverage domains. It can be seen that this fictional system would require a number of satellites, N_{th} given by

$$N_{\text{th}} = 16(R_p/d_c)^2(1 - \sin\lambda_{\text{cr}}) \tag{3}$$

where λ_{cr} is the critical latitude. Although meaningless as a physical system, Eq. (3) provides a convenient reference for comparing all practical systems. One may define ξ as the network efficiency factor:

$$\xi = 16(R_p/d_c)^2(1 - \sin\lambda_{\text{cr}})/N \tag{4}$$

which will provide a quantitative measure of the efficiency of any given practical system. By definition, the number of satellites required by a given system now is expressible in terms of that system's efficiency:

$$N = \frac{N_{\text{th}}}{\xi} = \frac{16(R_p/d_c)^2(1 - \sin\lambda_{\text{cr}})}{\xi} \tag{5}$$

As mentioned previously, a single example of a possible practical network now will be developed in order to provide a concrete example, both for purposes of clarity and for use as a numerical case in the final portion of this paper. This example system uses uniformly spaced polar satellitic orbits combined with hexagonal fire control patterns (or regions of responsibility). The hexagonal pattern (which appears to represent the most efficient physical possibility) and the corresponding satellite spacing pattern are illustrated in Fig. 4. It may be noted that two patterns actually are possible—either the one illustrated with the direction of the satellite orbits as shown, or the equivalent pattern with the satellite orbits existing 90° to those shown. The required number of satellites is identical in either case. Although there may be valid practical reasons for choosing one pattern over another, for the purposes of this investigation the case illustrated was chosen arbitrarily.

It may be seen that the longitudinal spacing between satellites in the space orbit is given by

$$d_{\text{long}} = 3^{1/2}d_c/2 \tag{6}$$

The lateral spacing between orbits at the critical latitude is given by

$$d_{\text{lat}} = 3d_c/4 \tag{7}$$

since d_{lat} is only the spacing between orbits at the critical latitude, the spacing between orbits at the equator must be found in order to determine the number of satellites required. Spherical geometry indicates that the spacing required at the equator is given by

$$d_{\text{orb}} = 3d_c/(4\cos\lambda_{\text{cr}}) \tag{8}$$

The total number of satellites required for a given diameter of the region of responsibility, d_c, now can be determined.

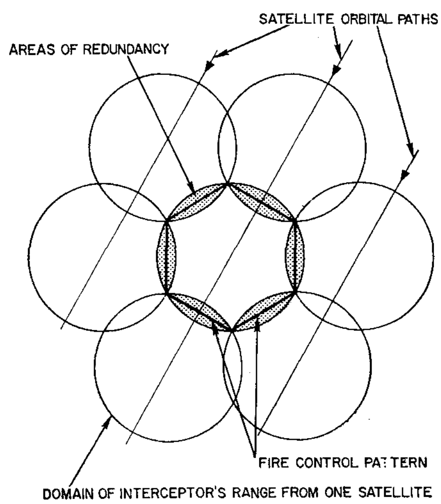

Fig. 4 Satellite responsibility and spacing for minimizing number of satellites

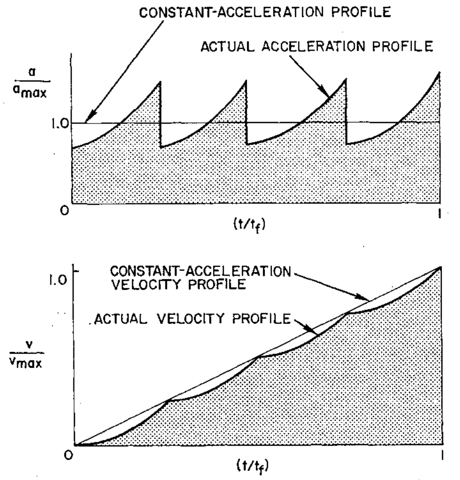

Fig. 5 Constant acceleration vs typical staged vehicle

The number of satellites per orbit is given by the circumference of the orbit divided by the spacing between satellites:

$$n(\text{per orbit}) = 4\pi R_p/(3^{1/2}d_c) \quad (9)$$

The total number of orbits is given by the circumference of one orbit divided by two times the spacing between orbits:

$$m(\text{orbits}) = 4\pi R_p \cos\lambda_{cr}/(3d_c) \quad (10)$$

Thus, the total number of satellites required in the system as a function of the orbital altitude, diameter of the coverage area, and critical latitude is given by the product of Eqs. (9) and (10):

$$N = mn = \left(\frac{16\pi^2}{3(3)^{1/2}}\right)\left(\frac{R_p{}^2 \cos\lambda_{cr}}{d_c{}^2}\right) \quad (11)$$

Quantitative comparison of Eqs. (3) and (11) will show that the efficiency of the illustrated system, as compared to the hypothetical ideal, is slightly greater than 52% for complete world coverage ($\lambda_{cr} = 0°$ latitude).

An admittedly cursory examination of other network spacing possibilities and limiting cases has been performed. Using the case of complete world coverage as a convenient efficiency comparison, it appears that the limiting efficiency factor obtainable with any physically real (not necessarily practical) system is 79%. Furthermore, this examination suggests that the best value for a reasonably practical system may be near 63%. Although no studies of AICBM networks have been found in unclassified literature, several known investigations of observation satellite networks are given in Refs. 1–3. When interpreted properly, these analyses have significance in the AICBM case.

3. *Interceptor range relations*

It is desired to express the slant range of the interceptor, relative to the satellite, as a function of the interceptor characteristic velocity and the several pertinent system parameters.

Quantitative examination of the exact relations expressing the interceptor range shows that the gravitational and centrifugal accelerations tend to cancel each other, and the required thrust acceleration is large with respect to their difference. Accordingly, it is reasonable to neglect these external acceleration effects upon the interceptor range, which results in a considerable reduction in complexity.

An important parameter is the choice of interceptor acceleration profile. This effect is accounted for by the coefficient α:

$$\alpha \equiv V_{ch}t_f \Big/ \iint_0^{t_f} a(t)dt \quad (12)$$

The meaningful limits of this parameter range from $\alpha = 1$ for an initial-impulse profile to $\alpha = 2$ for a constant-acceleration profile. A practical "impulsive" system actually may have a value somewhat greater than 1.0 because of the necessity for at least low-acceleration ability during post-impulse flight. Similarly, practical vehicles normally would have α-values slightly less than 2.0 because of the effect of staging. This latter effect is illustrated in Fig. 5, which shows the acceleration and velocity profiles for a vehicle with a finite number of symmetrical stages.

A quantitative measure of the efficiency of the interceptor guidance system is given by a factor ν. A perfect system would guide the interceptor along the ideal trajectory to the actual interception point. This would correspond to $\nu = 1.00$; all real systems will, of course, have efficiencies less than one. As an indication of practical values of efficiency, studies at Autonetics on similar AICBM systems have indicated that efficiencies of at least 80 to 95% can be obtained.

Accepting the assumptions offered and accounting for the two important factors described, the resultant general expression for range as a function of guidance efficiency, characteristic velocity, time of flight, and acceleration profile coefficient is found:

$$r = \nu V_{ch}t_f/\alpha \quad (13)$$

4. *Interceptor weight*

It now is necessary only to obtain the weight of the interceptor vehicle as a function of the interceptor range and the pertinent propulsion parameters in order to be able to write an expression for the total in-orbit weight of the complete AICBM system. A necessary condition for expressing the weight of the interceptor is that it must be a simple analytical expression in order to obtain an analytical optimization of the total weight of the system.

The problem arises of the number of stages to assume for the interceptor vehicle. If a two-stage interceptor were assumed arbitrarily, for example, this assumption would be unrealistic in the optimization for very short ranges or long ranges. A practical rocket vehicle design is one having a sufficiently large number of stages so that increasing the number of stages does not result in further substantial weight savings.

A means of handling this problem in closed analytical form has been developed previously by this writer. Space limitations do not permit the presentation of the derivation here, but the method followed will be discussed briefly and the results presented.

An analytical solution for the gross weight-to-payload ratio μ of a rocket vehicle with a given number of stages first is desired. If it is assumed that all stages of the rocket vehicle have equal values for specific impulse I_{sp}, characteristic velocity V_{ch}, and structural deficiency factor η, the derivation results in the following relationship for a vehicle of S stages:

$$\mu = \frac{W_0}{W_{pay}} = \left[\frac{\exp(V_{ch}/I_{sp}gS)}{1 - \eta[\exp(V_{ch}/I_{sp}gS) - 1]}\right]^S \quad (14)$$

It is undesirable, however, to use this equation in the optimization analyses for two reasons: 1) although the equation is in closed analytical form, it is not a simple expression; and 2) the problem of how to handle the number of stages still exists.

A fortunate circumstance is discovered, however, if one derives the solution as the number of stages approaches infinity. This derivation can be accomplished by expanding the denominator of Eq. (14) into a doubly infinite series and neglecting the higher-order product terms. The resultant infinite series is shown to be convergent, and the value of its sum can be determined. The resulting relationship for a vehicle with an infinite number of stages is given by

$$\mu_\infty = \exp[(1 + \eta)(V_{ch}/I_{sp}g)] \quad (15)$$

Thus, a much simpler analytical expression is determined for the interceptor weight. It can be shown that the infinitely staged vehicle described by Eq. (15) represents the minimum-weight rocket vehicle to provide a given characteristic velocity, all other factors being equal.

Since it is somewhat inconvenient to manufacture physically an interceptor vehicle with an infinite number of stages, the weight of an interceptor vehicle with a practical number of stages can be approximated suitably by multiplying the weight of the infinitely staged vehicle by an appropriate factor, e.g., multiplying by 1.2, assuming that a weight 20% greater than the absolute minimum is acceptable. An examination of numerical cases for two-, three-, and four-stage vehicles indicates that the use of the value of 20% seems to be a reasonable criterion. However, in the analysis, this constant is a parameter so that its specific value can be left to the discretion of the system designer.

The weight-to-payload ratio of a rocket vehicle with a "sufficiently large" number of stages will be given by

$$\mu = (1 + \Omega) \exp[(1 + \eta)(V_{\text{ch}}/I_{\text{sp}}g)] \tag{16}$$

Thus, a relatively simple analytical expression is obtained which automatically "provides" a suitable number of stages and therefore is ideal for application to the optimization analysis.

Combining Eqs. (13) and (16), one now can obtain the weight-to-payload ratio of the interceptor as a function of the acceptable weight penalty, vehicle deficiency factor, acceleration profile, interceptor range, guidance efficiency of the interceptor, I_{sp} of the fuel, and time of flight permitted:

$$W_0/W_{\text{pay}} = (1 + \Omega)\exp[(1 + \eta)(\alpha r/\nu I_{\text{sp}}gt_f)] \tag{17}$$

C. Clumped Target Distribution

1. *Introduction*

The analyses developed in the preceding discussion now may be combined to obtain an expression for the in-orbit weight of the complete AICBM system and to minimize this weight by determining the optimum system design for any given set of system parameters.

Section IIC presents the analyses for the case of clumped target distribution only. As explained earlier, the explicit effect of assuming a clumped target distribution is to imply that the number of interceptors per satellite is not a function of the satellite coverage diameter d_c for values in the neighborhood of the optimum solution.

This assumption implies that the area diameter of the clumps is small with respect to d_c and that the average distance between clumps is large with respect to d_c. Consideration of the economics of the ICBM vs AICBM "game" will show that from the opponent's viewpoint this type of distribution strategy is precisely what he would use in order to maximize the cost of this AICBM system for a given number of available target vehicles. In fact, the ultimate strategy would be to place every available ICBM in a single small clump, thus perfectly satisfying the assumptions made here. Naturally, other considerations would make this strategy impractical so that the attacker is forced to compromise on some minimum number of clumps which would be acceptable to him in practice.

2. *Limited fire control prediction capability*

The significance of prediction capability in determining satellite coverage domain has been discussed previously. The analyses immediately following are based upon the assumption of a limited prediction capability for fire-control (or target assignment) purposes. It includes the quantitative effect of providing any degree of prediction from no prediction to perfect prediction. The limiting case of perfect prediction is more convenient to analyze separately and is accomplished in a later section.

a. Total system in-orbit weight. The total weight of a single satellite and its interceptors is given by

$$W_{\text{sat}} = W_{s0} + (1 + \delta)[p(1 + \Omega)\mu_\infty W_{\text{pay}}] \tag{18}$$

where W_{s0} is the fixed weight of the satellite, δ accounts for the variable weight of the satellite (not including interceptor weight), p is the number of interceptors per satellite, and μ_∞ is the gross weight-to-payload ratio of an interceptor with infinite staging. This formulation is based on the assumption that it is reasonable to account for the total empty weight (without interceptors) of the satellite as being expressible by the combination of a minimum, or fixed, weight plus a variable weight that is linearly proportional to the total weight of the on-board interceptors.

Substituting from Eq. (17) for μ(the gross weight-to-payload ratio), the total weight of one satellite as a function of the basic parameters is obtained:

$$W_{\text{sat}} = W_{s0} + (1 + \delta)(1 + \Omega)pW_{\text{pay}} \times \exp[(1 + \eta)(\alpha r/\nu I_{\text{sp}}gt_f)] \tag{19}$$

Combining Eqs. (2, 5, and 19), one finally obtains Eq. (20), representing the expression that must be minimized with respect to the range of the interceptor:

$$W_{\text{sys}} = NW_{\text{sat}} = \left[\frac{4R_p{}^2(1 - \sin\lambda_{\text{cr}})}{\xi[(r^2 - h^2)^{1/2} - (\Lambda R_t)^2]}\right] \cdot \left\{W_{s0} + (1 + \delta)(1 + \Omega)\; pW_{\text{pay}} \exp\left[(1 + \eta)\left(\frac{\alpha r}{\nu I_{\text{sp}}gt_f}\right)\right]\right\} \tag{20}$$

For manipulative convenience, new symbols are defined for a number of the parameters in Eq. (20). These are constant with respect to the relative range r. The following equation for the system orbital weight now may be written from Eq. (20) by definition:

$$W_{\text{sys}} \equiv \frac{\psi_1}{\{[r^2 - h^2]^{1/2} - \Lambda R_t\}^2} \{W_{s0} + \psi_2 e^{\alpha\phi r}\}$$

where

$$\psi_1 \equiv \left[\frac{4R_p{}^2(1 - \sin\lambda_{\text{cr}})}{\xi[(r^2 - h^2)^{1/2} - (\Lambda R_t)^2]}\right]$$
$$\psi^2 \equiv [(1 + \delta)(1 + \Omega)pW_{\text{pay}}] \tag{21}$$
$$\phi = [(1 + \eta)/\nu I_{\text{sp}}gt_f]$$

b. Investigation of limiting cases. Examination of the limits of Eq. (21) will provide 1) proof that an optimum value of interceptor range exists, 2) a better appreciation of the trade-offs that occur in the optimization, and 3) a priori knowledge of a trivial solution to the later-developed extremal relationship.

1) Coverage domain approaches zero. For the first limiting case, let d_c approach zero. This relation represents the minimum range that the interceptor may have, since any smaller range would provide a physically meaningless value for the diameter of coverage. From Eq. (2), the expression then can be written as

$$d_c = \{[r^2 - h^2]^{1/2} - \Lambda R_t\} \to 0 \tag{22}$$

Examination of Eq. (21) will show that the case represented by Eq. (22) will cause the weight of the system to approach infinity because the number of satellites will approach infinity while the weight of each satellite will remain finite. Thus, a minimum value for the relative range of the interceptor is determined:

$$r_{\min} = [(\Lambda R_t)^2 + h^2]^{1/2} \qquad \text{yielding } W_{\text{sys}} \to \infty \tag{23}$$

and it is shown that this minimum range results in an infinite orbital system weight.

2) Interceptor range approaches infinity. The other extreme is that the relative range of the interceptor be permitted to approach infinity. To examine the significance of this limiting case, one may use L'Hospital's rule to evaluate the ambiguous limit as r approaches infinity. The final result is

$$\underset{r\to\infty}{\mathcal{L}}[W_{sys}(r)] = \underset{r\to\infty}{\mathcal{L}} \frac{1}{2}\psi_1\psi_2(\alpha\phi)^2 e^{\alpha\phi r} \to \infty \tag{24}$$

Thus, the total weight of the system also is seen to approach infinity as the interceptor range increases without limit.

3) Intervening values of interceptor range. Finally, examining Eq. (21) for values of the relative range which lie between the two limiting values, i.e.,

$$[(\Delta R_t)^2 + h^2]^{1/2} < r < \infty \tag{25}$$

it is clear that the total system weight is finite. Thus, the existence of an optimum value of the interceptor relative range, which will minimize the total in-orbit weight, has been proved.

c. *Optimization analysis*. With the development of Eq. (21) completed and the existence of an optimum solution proven, one may proceed now to the determination of the extremal relationship that will provide the minimum weight system as a function of the various design parameters and constraints.

Taking the partial derivative of the weight of the system with respect to the interceptor relative range and forcing the resulting relationship to zero, one obtains

$$\frac{\partial}{\partial r}(W_{sys}) = \frac{\partial}{\partial r}\left[\frac{\psi_1\{W_{s0} + \psi_2 e^{\alpha\phi r}\}}{\{[r^2 - h^2]^{1/2} - \Delta R_t\}^2}\right] = 0 \tag{26}$$

Performing the indicated partial differentiation,

$$\frac{\partial}{\partial r} W_{sys} = \frac{\{[r^2 - h^2]^{1/2} - \Delta R_t\}^2\{\psi_1\psi_2(\alpha\phi)e^{\alpha\phi r}\} - 2\psi_1\{W_{s0} + \psi_2 e^{\alpha\phi r}\}\{1 - [\Delta R_t/(r^2 - h^2)^{1/2}]\}r}{\{[r^2 - h^2]^{1/2} - \Delta R_t\}^4} = 0 \tag{27}$$

Examination of Eq. (27) indicates that there are only two means by which the expression can be equal to zero: 1) if the denominator is infinite, or 2) if the numerator is equal to zero. The possibility that an infinite denominator would represent the optimum may be ignored, since that would require the relative range to be infinite which already has been shown to yield an infinite system orbital weight rather than a minimum.

Hence, the extremal relationship can be obtained by setting the numerator equal to zero. The optimum value of interceptor range must be one of the roots of the resulting relationship:

$$\{[r^2 - h^2]^{1/2} - \Delta R_t\}^2\{\psi_1\psi_2(\alpha\phi)e^{\alpha\phi r}\} = 2\psi_1\{W_{s0} + \psi_2 e^{\alpha\phi r}\}\{1 - [\Delta R_t/(r^2 - h^2)^{1/2}]\}r \tag{28}$$

Careful examination of Eq. (28) does not indicate any analytical solution in the present form. Fortunately, however, it is not unreasonable to assume that the fixed weight of the satellite will be small with respect to the total variable satellite weight, i.e., with respect to the total weight of interceptors plus the variable weight of the satellite itself.

If this assumption is accepted, it is found that Eq. (28) will reduce to a quartic polynomial. The following relations result.

Assuming

$$W_{s0} \ll \psi_2 e^{\alpha\phi r} \tag{29}$$

then,

$$(W_{s0} + \psi_2 e^{\alpha\phi r}) \approx \psi_2 e^{\alpha\phi r} \tag{30}$$

The optimization relationship now is given by

$$\{[r^2 - h^2]^{1/2} - \Delta R_t\}^2\{\psi_1\psi_2\alpha\phi e^{\alpha\phi r}\} = 2\psi_1\{\psi_2 e^{\alpha\phi r}\}\{1 - [\Delta R_t/(r^2 - h^2)^{1/2}]\}r \tag{31}$$

Physical considerations make it apparent that

$$e^{\alpha\phi r} = 0 \tag{32}$$

is a meaningless relationship because it implies that either $\alpha\phi$ or r must be equal to negative infinity. Furthermore,

$$\psi_1 = 0 \qquad \psi_2 = 0 \tag{33}$$

both are trivial solutions to the relationship. In addition, it has been shown previously that

$$\{[r^2 - h^2]^{1/2} - \Delta R_t\} = 0 \tag{34}$$

yields a maximum system weight rather than a minimum solution. Hence, the extremal relationship may be simplified without loss of nontrivial solutions by dividing through by

$$\psi_1\psi_2 e^{\alpha\phi r}\{[r^2 - h^2]^{1/2} - \Delta R_t\} \tag{35}$$

Dividing Eq. (28) by the quantity in Eq. (35) and performing a number of algebraic manipulations, the following polynomial in r can be obtained:

$$r^4 + \left[-\frac{4}{\alpha\phi}\right]r^3 + \left[-2h^2 + \frac{4}{(\alpha\phi)^2} - (\Delta R_t)^2\right]r^2 + \left[\frac{4h^2}{\alpha\phi}\right]r + [+h^4 + (\Delta R_t)^2 h^2] = 0 \tag{36}$$

A closed-form analytical solution to this general fourth-degree polynomial apparently can be obtained, but it appears that the solution would be sufficiently tedious and complex to require numerical evaluation of its parameters in any event. It therefore appears desirable to obtain a numerical solution by the use of a digital computer program for the solution of general quartics.

In obtaining a numerical solution, considerable effort can be avoided by reducing the parameters to the nondimensional pi-parameters associated with the Buckingham pi theorem. This theorem tells one that the relationship in Eq. (36) can be described completely by three pi-terms, and a numerical solution then can be found as a function of only two parameters. Thus, the total number of computations required is reduced by a substantial factor.

A convenient formulation of pi-terms can be found which has the advantage of a meaningful solution when (ΔR_t) approaches zero. The resulting optimization relationship is

$$(\alpha\phi r)^4 + [-4](\alpha\phi r)^3 + [-2(\alpha\phi h)^2 + 4 - (\alpha\phi\Delta R_t)^2](\alpha\phi r)^2 + [4(\alpha\phi h)^2](\alpha\phi r) + [(\alpha\phi h)^4 + (\alpha\phi\Delta R_t)^2(\alpha\phi h)^2] = 0 \tag{37}$$

This, then, is the final resulting extremal relationship. The solution for the roots of $(\alpha\phi r)$ of this equation will represent the optimum values for interceptor relative range as a function of all of the parameters involved in the defined variables.

As a convenience in computation and as a check on the general solution, it is interesting to note that an analytical closed-form solution is obtained when one lets the value of the parameter h, the relative altitude-change of the interceptor vehicle, be small with respect to the optimum range of the vehicle:

$$(h)^2 \ll (r)_{opt}^2 \tag{38}$$

Substituting this relationship into Eq. (37), the quartic polynominal reduces to a simple quadratic:

$$(r)^2 + [-4/\alpha\phi]r + \{+ [4/(\alpha\phi)^2](\Delta R_t)^2\} = 0 \quad (39)$$

Thus, the optimum value of the relative range can be obtained in simple form for this special case:

$$r_{\text{opt}} = (2/\alpha\phi) \pm (\Delta R_t) \quad (40)$$

The positive sign preceding R_t can be chosen from physical considerations. Rewriting Eq. (40) in terms of the original variables and using the pi-parameters, one obtains the final relationship:

$$(\alpha\phi r_{\text{opt}}) = +2 + \alpha\phi\Delta R_t$$

or

$$\alpha[(1 + \eta)/\nu I_{\text{sp}} g\ t_f]r_{\text{opt}} = 2 + [\alpha(1 + \eta)/\nu I_{\text{sp}} g\ t_f]\Delta R_t \quad (41)$$

The general extremal relationship, Eq. (37), has been solved on an Autonetics' Recomp II Computer and combined with the values obtained from the special case solution of Eq. (41). The solutions are illustrated in nondimensional parametric form in Fig. 6a.

3. *Perfect fire-control prediction*

A similar determination of total system in-orbit weight and optimization of interceptor range can be developed when it is assumed that perfect fire-control prediction is available. This is, of course, the extreme case because perfect prediction never can be expected. However, by examining the two extremes of no prediction and perfect prediction, one's judgment can be used to determine the effect of prediction within a given degree of inaccuracy; the effect can be determined analytically if the proper value for Λ is obtained for any given prediction concept.

It will be recalled from Eq. (2) that Λ is a parameter that is dependent upon the assumption of fire-control prediction, and when perfect prediction is assumed, Λ is equal to zero. Examination of the relationships existing in the previous section (where Λ was assumed not equal to zero) will show that no manipulations that created meaningless steps in the solution or that lost nontrivial solutions were performed.

Therefore, it is possible simply to let Λ equal zero in the optimization relationship of Eq. (28). When this substitution is made, one obtains the new relationship:

$$[r^2 - h^2][\psi_2\alpha\phi e^{\alpha\phi r}] = 2[W_{s0} + \psi_2 e^{\alpha\phi r}]r \quad (42)$$

Once more a closed-form analytical solution cannot be found without making the assumption that the fixed weight of the satellite is small with respect to its variable weight. Accepting this relationship, the solution reduces to the simple form

$$[r^2 - h^2][\alpha\phi] = 2r \quad (43)$$

Solving for the optimum value of the relative range from this equation yields the solution

$$r_{\text{opt}} = +(1/\alpha\phi) + \{h^2 + [1/(\alpha\phi)^2]\}^{1/2} \quad (44)$$

A negative value for the square root term will yield a negative solution for r, and so it is clear from the physical considerations that the proper sign is positive.

The Buckingham pi theorem shows that one can reduce this solution to two pi-terms:

$$(\alpha\phi r_{\text{opt}}) = 1 + [(\alpha\phi h)^2 + 1]^{1/2} \quad (45)$$

This expression then yields the optimum values for the relative range when it is assumed that perfect fire-control prediction is used. Figure 6b shows graphically the solutions to this equation.

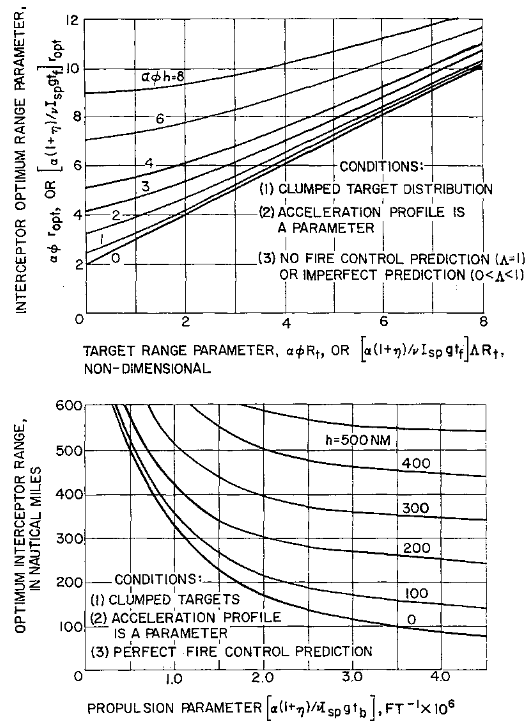

Fig. 6 Optimum interceptor ranges, clumped target distribution: a) with no prediction or limited prediction; b) with perfect prediction

D. Homogeneous Target Distribution

When the potential target ICBM's are assumed to be distributed throughout some large critical region (i.e., large with respect to the interceptor range capability) in a homogeneous pattern, and it further is assumed that the targets are launched in a uniform random fashion with respect to time, the most reasonable assumption for the number of satellites per interceptor is that the number should be directly proportional to the area of responsibility of the satellite.

A similar system analysis and optimization solution for this case has been performed by this writer. In the interests of brevity, that case is not presented here but can be found in Ref. 4.

Suffice it to comment here that 1) the clumped distribution assumptions seem much closer to the real circumstances, and 2) the clumped distribution definitely represents the most difficult AICBM problem. The homogeneous case thus serves primarily as an interesting comparison to indicate the sensitivity of the optimizations to variations in assumed target distribution. Interestingly, the optimum interceptor range, for reasonable estimates of target density, proves to be of the same order as that for a typical clumped case.

III. Some Numerical Applications of the Analyses

The nondimensional parameters presented in the preceding section have the advantage of maximum flexibility and provide a convenient presentation from which an infinite variety of combinations of comparisons can be made. It is difficult, however, to appreciate fully the significant results of the optimization without examining some numerical examples of applications of these relationships.

This section, therefore, chooses specific values for the various parameters in order to present numerical results for these particular cases. Although they are believed to be realistic, it should be emphasized that most of these numerical values are chosen somewhat arbitrarily and, therefore, should be considered only as a reasonable example of the application of the curves.

Table 1 Numerical values assumed

Symbol	Parameter[a]	Assumed value
R_t	Range of target	1.5×10^6 ft (247 naut miles)
η	Interceptor deficiency factor	0.15
ν	Guidance efficiency factor	0.90
I_{sp}	Specific impulse of propellant	330 lb-sec/lb
g	Acceleration of gravity	32.2 ft/sec^2
t_f	Maximum interceptor time-of-flight	120 sec
h_{rel}	Maximum vertical travel of interceptor	1.0×10^6 ft (165 naut miles)
ξ	Satellite network efficiency factor	0.52

[a] Symbols are defined more fully in the Nomenclature.

A. Assumed Numerical Values

Table 1 presents the numerical values of the parameters which are, in general, assumed in this section. Note that the satellite network implied by ξ is the example system described previously under "Satellite Orbital Networks."

Table 1 is presented without further comment except for the following discussion on the basis for the assumptions on the range of the target during the intercept flight time and the time of flight permitted for the interceptor.

Study of the satellite-based interceptor concept will indicate that the system that is designed to handle more than one type of target will have its permitted time of flight of the interceptor determined by the shortest flight time of any of the potential targets. The range of the target, which must be considered in these equations, is the maximum range that any of the targets may have during the permitted time of interception. Thus, in general, the highest acceleration target will be the design criterion.

For this presentation, therefore, the Minuteman vehicle makes an ideal sample target because 1) it represents a rather difficult type of target for the system, and 2) sufficient unclassified data are available for the purposes of this examination. Released unclassified Minuteman data include sufficient information to make a crude estimate of boost range vs time sufficient for the present purposes. These data include information that the total boost period is approximately 3 min. From the total of 180 sec of boost time, one must allow (in the AICBM system) a reasonable period of time for 1) the sighting of the target, 2) any computations that are necessary, and 3) the ignition and release of the interceptor. Furthermore, one must assume that the interceptor encounters the target vehicle at a reasonable length of time prior to the end of the boost phase in order to perform its kill successfully. With these thoughts in mind and the data available, it appears that a time of interceptor flight of about 120 sec and a range of target of about 1.5×10^6 ft (approximately 250 naut miles) are reasonable numbers to use for the present purpose.

B. Comparison of Optimized Systems

Table 2 presents a comparison of numerous details of the various discussed system concepts. Each data entry in this chart represents the optimum value for that particular parameter based upon the optimization analysis. Thus, with the use of this table (and similar tables for other numerical data), one has a reasonable quantitative basis for making comparisons and judgments between the various systems in that each system has been optimized independently before the comparison is made.

Table 2 presents a number of pertinent parameters, such as optimum relative range and maximum absolute range that the interceptor may travel, optimum characteristic velocity of the interceptor, average acceleration, gross weight-to-payload ratio of the interceptor, number of satellites which will be required in the complete orbital system for worldwide coverage, diameter of the coverage domain, and, finally, the parameter that represents the total in-orbit weight of the anti-ICBM system. The last parameter presented is the ratio of the total in-orbit weight to the product of the number of interceptors per satellite times the payload weight of each interceptor vehicle. Thus, this latter column is directly proportional to the total in-orbit weight for a given interceptor payload and for any chosen number of interceptors per satellite (which should be recalled as assumed to be not a function of range in the clumped target distribution case).

The systems now can be compared realistically to determine the quantitative benefits of prediction and high early-acceleration. One may compare rationally the advantages and disadvantages that may exist for each system.

The numerical data used in Table 2 are listed below the table and are the same as those indicated previously, except for the addition of a 20% finite interceptor staging factor and a 20% satellite variable weight factor. These two items have no effect on the optimization but directly affect the total system weight.

Careful examination of Table 2 will provide considerable insight into the ranges of values of the pertinent parameters and into the effects of prediction and acceleration profile. Depending upon the system concept, the values of optimum effective interceptor relative range are seen to vary from 270 to 600 naut miles and the characteristic velocities from roughly 22,000 to 50,000 fps. The optimum number of satellites for 100% global coverage varies from 810 to 3620.

By far the most pertinent and striking comparison presented, however, is the measure of total in-orbit weight. This parameter is seen to vary by up to two orders of magnitude, from 128,000 to 1,280,000. [Note that the application of estimates of 1) required interceptor payload weight, 2) a reasonable number of interceptors per satellite, and 3) cost to place 1 lb in orbit will convert these numbers into estimates of the total cost for boosting one complete AICBM system into orbit.]

Caution must be taken, however, not to assume that this parameter is proportional identically to total in-orbit weight. In the cases of impulsive acceleration, for example, two factors can cause a significant increase in weight: 1) increases in structural requirements of vehicle and payload due to

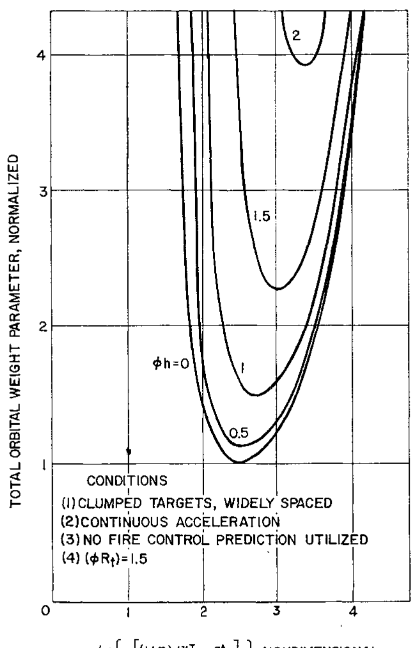

Fig. 7 Sensitivity to nonoptimum design

Table 2 Numerical-example data for several optimized systems[a]

CONCEPT: Acceleration	Prediction	Relative Range r_{rel} (naut mi)	Maximum Absolute Range (r_{abs}) max (naut mi)	Characteristic Velocity V_{ch} (ft/sec)	Average Acceleration (g's)	Interceptor Weight $\frac{W_o}{W_{pay}}$	Diameter of Coverage (naut mi) d_c	Number of Satellites N	Orbital Weight of System $\frac{W_{sys}}{pW_{pay}}$
Constant	None	453	951	50,900	13.1	294	344	3620	1,280,000
Constant	Perfect	270	768	30,400	7.86	32.3	427	2290	88,800
Impulsive	None	600	1098	33,800	N/A	46.5	658	970	54,000
Impulsive	Perfect	395	893	22,200	N/A	13.2	718	810	12,800

[a] Conditions: 1) clumped targets, widely spaced; 2) maximum relative altitude travel, h = 165 naut miles; 3) guidance efficiency, ν = 0.90; 4) propellant average specific impulse = 330 lb-sec/sec; 5) interceptor deficiency factor of interceptor, η = 0.15; 6) interceptor time of flight, t_f = 120 sec; 7) target horizontal travel during t_f, R_t = 250 naut miles; 8) interceptor finite stages factor, Ω = 0.20; 9) satellite weight factor, δ = 0.25.

high-acceleration boost, and 2) the possibility that a substantial amount of "correction" propellants must be added to the "payload" of the final stage, since the correction propellant could not be assumed to add to the range of coverage.

These cautions notwithstanding, it is clear nevertheless that tremendous benefits are available to the system design that can use successfully high-acceleration initial boost and a reasonable degree of prediction.

C. Nonoptimum System Comparison

An always appropriate question in any optimization study is, what is the sensitivity to the optimum values? Although a true minimum is recognized to occur, is the effect on the final results really sufficient to cause concern? In order to indicate the sensitivity of the satellite-based AICBM system to the optimum values, Fig. 7 presents the total system in-orbit weight where the parameter ϕR_t equal to 1.5 is used as an example.

It is seen that, in fact, the total system weight is rather sensitive to the optimum values. For example, for the case $\phi h = 1$, the minimum weight occurs when (ϕr) is 2.75, and it is seen that the total system weight has an increase of 100% if a range of 40% above, or 20% below, the optimum value is used. As with any zero-slope optimization, of course, there is a finite region in the neighborhood of an optimum value where the optimized parameter is not affected too greatly by variations of the optimum value. By using curves similar to those in Fig. 7 and plotting in the immediate vicinity of the previously determined optimum values, one can determine the range of the effective region available for perturbing the optimum values to account for factors not included in the optimization analyses.

IV. Conclusion

A number of possible satellite-based AICBM system concepts have been analyzed and optimized independently in this paper. Both analytical and graphical optimization results have been presented in general parametric form. Several numerical example applications of the parametric solutions also have been provided for clarity. In addition to the optimization values, the most pertinent results presented are the comparison of the total weight in orbit when fire-control prediction and impulsive acceleration are used. The potential benefits of these factors are found to be significant. In particular, considerable effort would be justifiable to obtain some degree of fire-control prediction.

Many of the numerical data presented herein are compared at the extreme limits of possibility, i.e., perfect prediction vs no prediction, impulsive vs constant acceleration, and clumped vs homogeneous target distribution. It is important to recognize, however, that the general equations developed provide parametric means for quantitatively examining the intervening parameter values in the first two cases. In the third case (target distribution), two factors should be kept in mind: 1) it appears very likely that the actual distributions would approach closely those defined for the clumped distribution, and 2) one easily may use reasonable judgment to extrapolate between the two cases if this is desired.

Naturally, certain practical factors always exist which have some effect on an overall optimization and yet were not possible to include in the optimization analysis. Three such factors worth mentioning are 1) the effect of not neglecting the "fixed" weight of the satellites, 2) a maximum limitation on the range of the satellite and interceptor sensors, and 3) the possibility that higher acceleration levels could be required in some cases to overcome maneuver effects of high-acceleration target vehicles. The first two factors tend to decrease the otherwise desired range, whereas the third factor tends to increase the range. Factors such as these, of course, always can be treated in numerical form when a specific design is considered. Because of the indicated sensitivity of the system to the optimum values determined, it appears unlikely that these or other factors will have a very great effect upon the optimum values.

In conclusion, the reader is cautioned to use care in comparing the results of one of the presented basic concepts with another to be sure that one of the concepts may not imply a relative change in payload weight, structural deficiency, or other factors not accounted for automatically in the analyses. If the results are applied with reasonable judgment, it is believed that they can provide a valuable aid in designing and comparing various system concepts and parameters in a truly realistic manner.

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